Open University Psychological Society presents

The Beginner's Guide to Quantum Cognition

A talk by Dr. Rachael Bond

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This document is a copy of the presentation used during a lecture delivered to the Open University Psychological Society in Brighton, UK, in March, 2019.

Brief explanatory notes are also included.

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Is the
brain a
quantum
computer?

- Although large scale quantum computers have yet to be built, there are many algorithms which demonstrate their capabilities.
- For instance, the breaking of RSA encryption, widely used on the internet, uses Shor’s algorithm.
- Shor’s algorithm factors large numbers in to 2 unique prime numbers.
- A quantum computer can also use Grover’s algorithm to select the correct card in a game of “Four-card Monte” on the first try.
- A classical computer simulating human search patterns would take, on average, 2½ tries to turn over the correct card.
- There is a demonstration of the quantum Four-card Monte game on the IBM website at https://www.research.ibm.com/ibm-q/quantum-card-test/
- These are things that humans simply cannot do.

A Partial List of Quantum Computing Algorithms

Aharonov-Jones-Landau algorithm

Amplitude amplification

BHT algorithm

Deutsch-Jozsa algorithm

Grover's algorithm

Hadamard transformation

Hidden subgroup problem

Quantum annealing

Quantum Fourier transformation

Quantum phase estimation algorithm

Quantum walks

Shor's algorithm

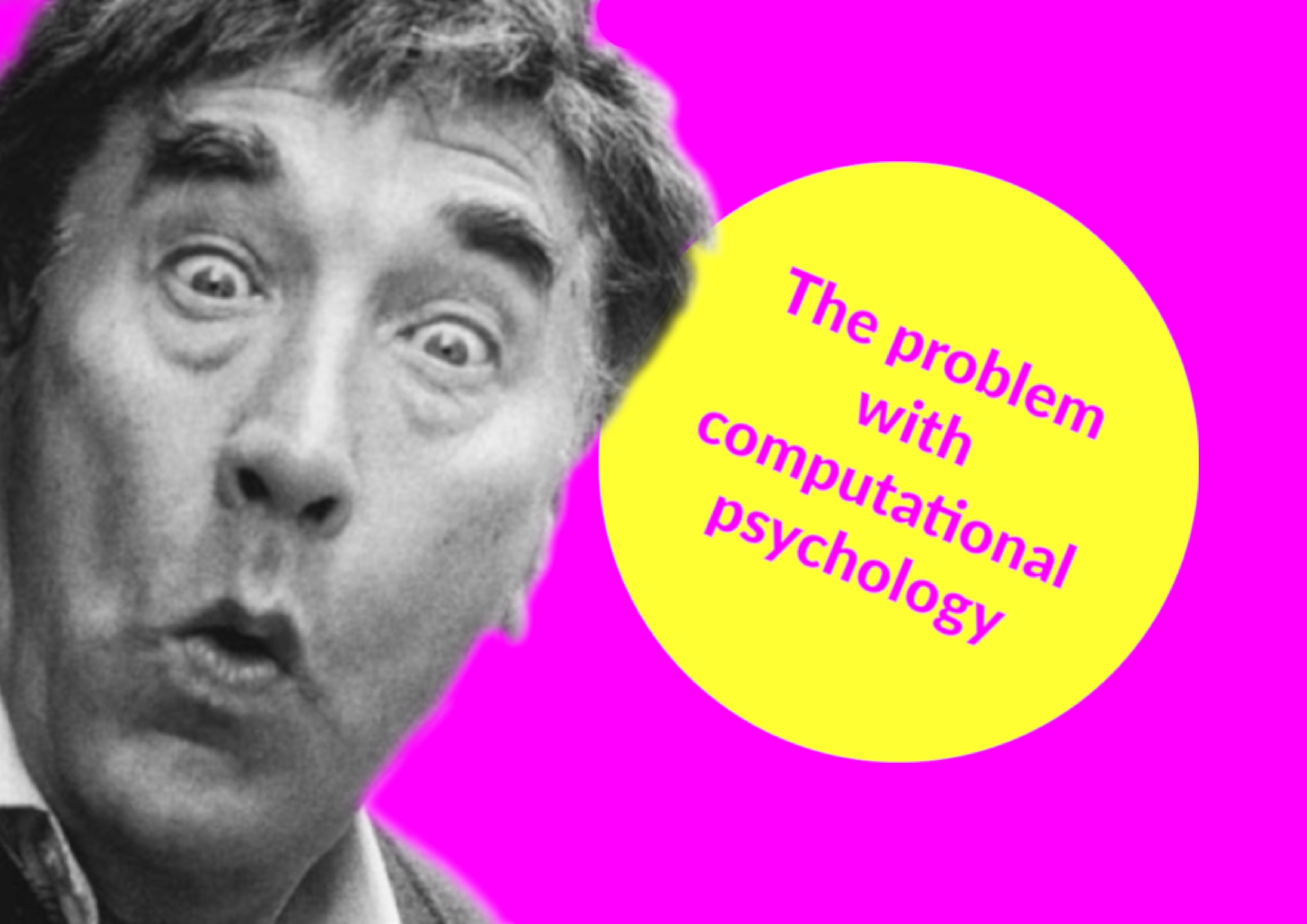
Factoring Large Numbers

2799783391 1221327870 8294676387 2260162107
0446786955 4285375600 0992932612 8400107609
3456710529 5536085606 1822351910 9513657886
3710595448 2006576775 0985805576 1357909873
4950144178 8631789462 9518723786 9221823983

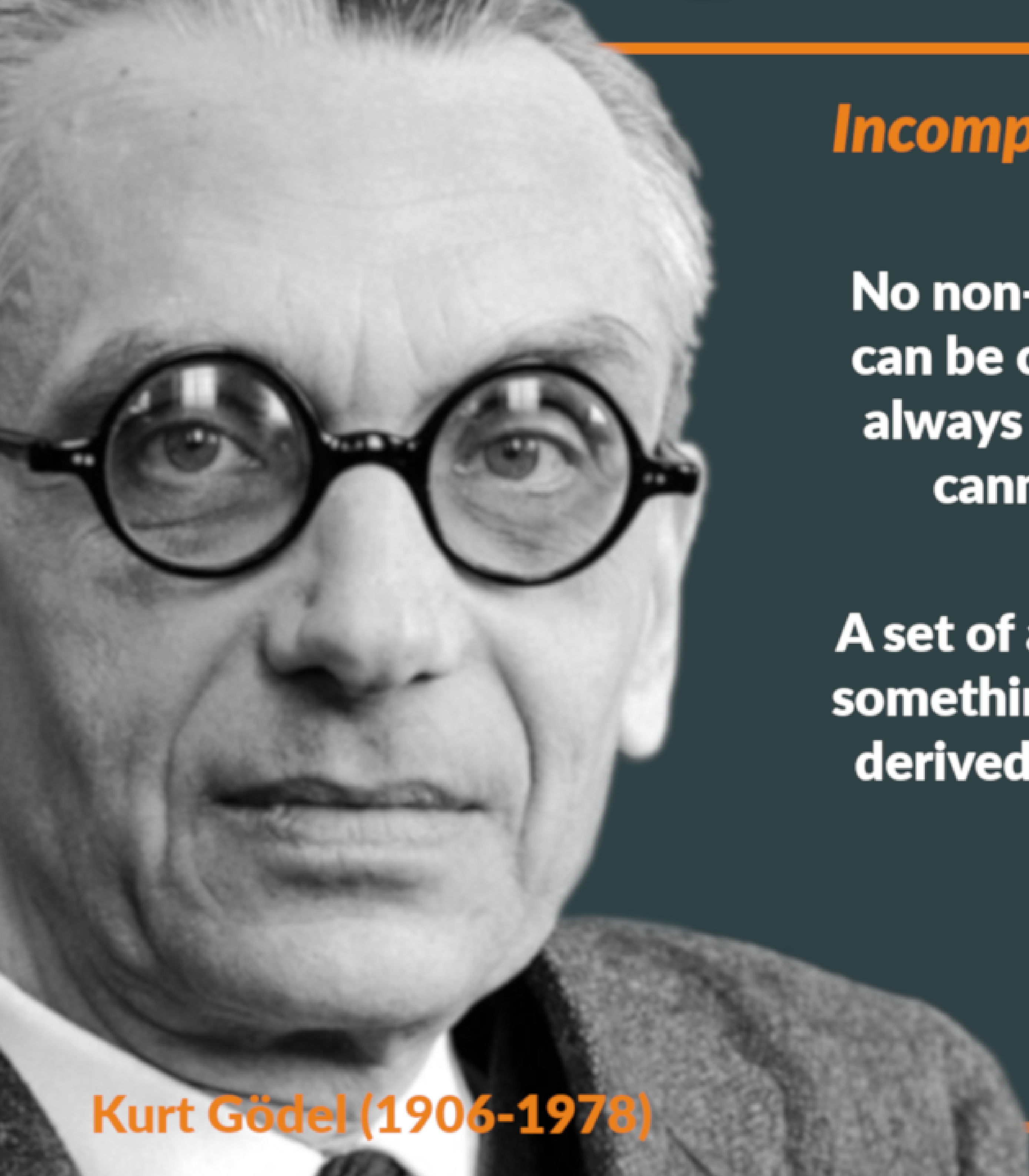
uniquely factors to these 2 prime numbers

3532461934 4027701212 7260497819 8464368671
1974001976 2502364930 3468776121 2536794232
0005854795 6528088349 * 7925869954 4783330333
4708584148 0059687737 9758573642 1996073433
0341455767 8728181521 3538140930 4740185467


4-card monte: Find the Queen (Classical computer)
(0)
(0)
(1)
(0)
Encode the cards as bits:
(0) for NOT Queen; (1) for IS Queen.
4-card monte: Find the Queen (Classical computer)
(0)
(?)
(?)
(?)
Encode the cards as bits:
(0) for NOT Queen; (1) for IS Queen.
Sequentially measure each bit until (1) is returned.
4-card monte: Find the Queen (Classical computer)
(0)
(0)
(?)
(?)
Encode the cards as bits:
(0) for NOT Queen; (1) for IS Queen.
Sequentially measure each bit until (1) is returned.
4-card monte: Find the Queen (Classical computer)
(0)
(0)
(1)
(?)
Encode the cards as bits:
(0) for NOT Queen; (1) for IS Queen.
Sequentially measure each bit until (1) is returned.
On average it will take 2½ attempts to return (1).


The problem with computational psychology

- McCulloch and Pitts first suggested the computational model of cognition in 1943. [doi: 10.1007/BF02478259]
- The most obvious product of the computational model of cognition is neural-nets.
- However, there are differences between neural-nets and human cognition.
- Neural-nets require extensive teaching to perform a task, whereas humans are able to self-direct learning.
- Human brains contain feedback, as well as feed-forward, loops. In neural-nets feedback loops create mathematical instability.
- Such differences may be because neural-nets were designed to mimic the physical brain rather than cognitive processes.
- There is little evidence that cognition is algorithmic.
- That is not to say that humans can’t think and reason to algorithmic rules, only that there also appear to be non-algorithmic processes.
- Non-algorithmic cognition may include non-monotonic logic, where unexplainable conceptual or logical leaps are made.
- Abductive logic, where contextual knowledge or beliefs may override a purely syntactic evaluation of data, is also an example of non-algorithmic cognition.



Kurt Gödel (1906-1978)

Incompleteness theorem

No non-trivial set of axioms can be complete - there will always be questions which cannot be answered.

A set of axioms cannot prove something true that has been derived from those axioms.

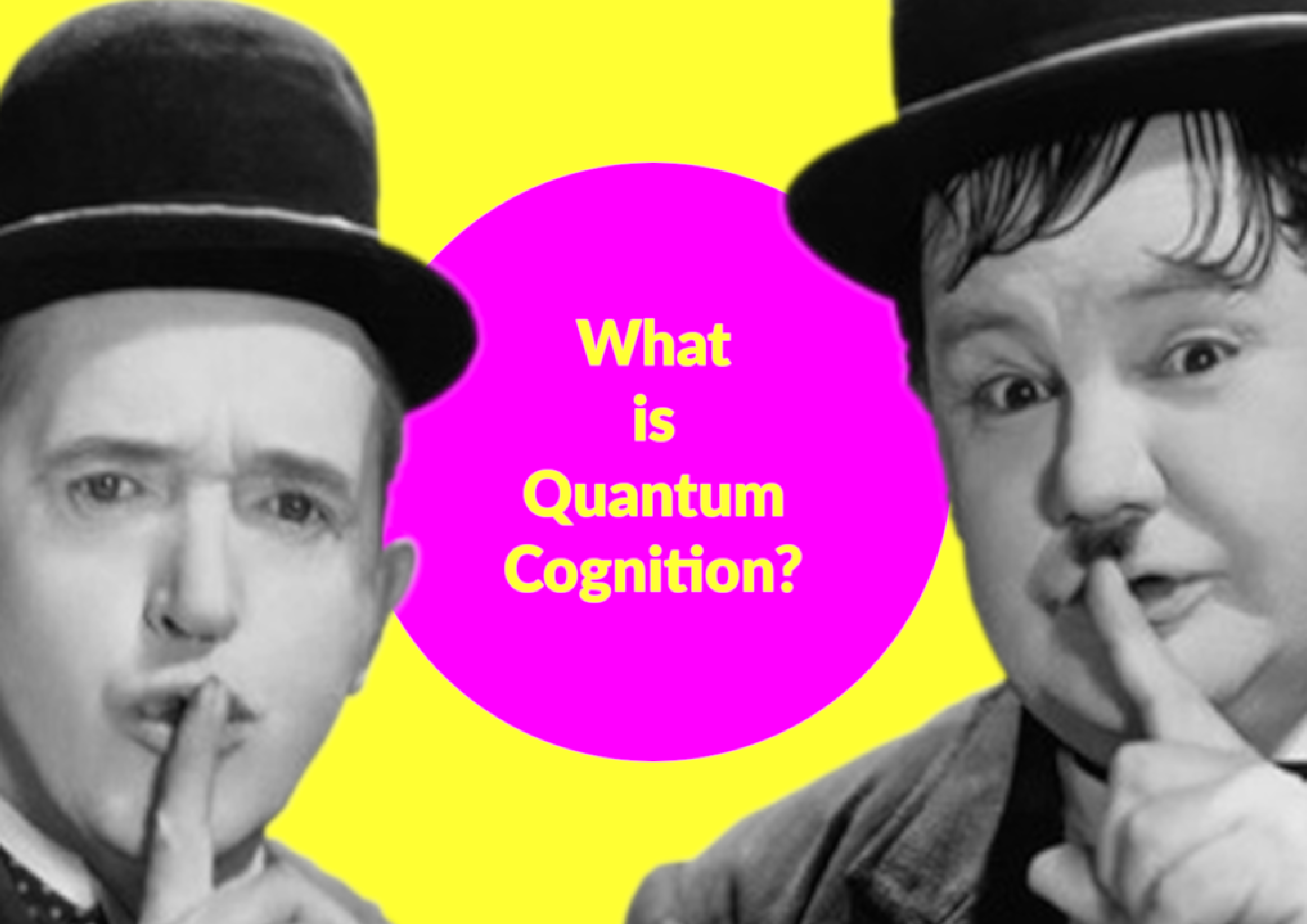
- Kurt Gödel created the “Incompleteness Theorem”.
- The Incompleteness Theorem says that within any non-trivial set of rules there will always be questions that cannot be answered, and that no rules-based system can ever be complete.
- Because of this, the rules by which something is discovered cannot also be used to demonstrate universal correctness — that would be a tautology.

" I believe there is something going on in a conscious being ... that is not a computational activity.

And to be conscious at all is not a quality that a computer as such will ever possess - no matter how complicated, no matter how well it plays chess or any of these things. "

Sir Roger Penrose (1931-)

- Roger Penrose’s work has been strongly influenced by people such as Gödel and Escher.
- In his 1989 book, “The Emperor’s New Mind”, Roger Penrose argues that human cognition cannot be described algorithmically.
- Roger Penrose believes that the brain is a deterministic, non-algorithmic system.
- “Deterministic” because consciousness has to be created by something.
- “Non-algorithmic” because it cannot be mathematically modelled.
- Roger Penrose argues that consciousness is the result of quantum gravity effects in microtubules, which form part of the cytoskeleton of cells.
- Max Tegmark, however, has calculated that neurons fire 10,000 million times too slowly to be compatible with quantum decoherence [arXiv:quant-ph/9907007].
- Because Roger Penrose’s model is specifically non-algorithmic it is very difficult to test empirically.


What
is
Quantum
Cognition?

- Quantum Cognition is a “normative” research approach in Psychology.
- The starting point for normative research is a mathematically correct model of how a task should be done.
- Because the models are mathematical, or statistical, they lend themselves to research in decision-making theory and game-play theory.
- In empirical research, instead of having 2 groups of participants it is common for the mathematical model to take the place of one of the groups.
- Any differences in the approaches of the human participants to that demanded by the mathematical model gives insight into the nature of cognition.
- However, for this to be a robust research approach, the mathematical model must definitely be correct — and quantum mathematics often gives different solutions to classical mathematics.
- I would define Quantum Cognition as “a generalised mathematical approach for the normative modelling of cognitive processes, and is derived from the principles of quantum mechanics”.
- No claim is made that there are quantum mechanical processes in the brain.

Quantum Cognition: A Definition

Quantum Cognition is a generalised mathematical approach for the normative modelling of cognitive processes, and is derived from the principles of quantum mechanics.

No claim is made that quantum processes play any role in human cognition.

- Quantum mechanics uses a different set of rules (the “axiomatic base”) to classical mathematics.
- Quantum mechanics uses the von Neumann axioms and matrix mathematics, rather than classical Kolmogorov axioms.
- Because of this, the mathematics of quantum mechanics is “non-commutative” which means that A*B is usually not equal to B*A.
- The von Neumann axioms include the Kolmogorov axioms as a special case, so they are the most general version of mathematics.

Maths

2 x 3 = 6

3 x 2 = 6

Is it always true that
2 x 3 = 3 x 2 ?

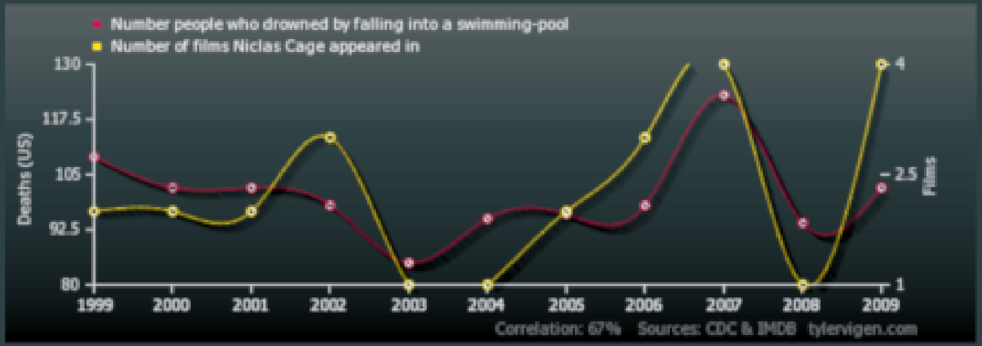
No, because if you multiply matrices

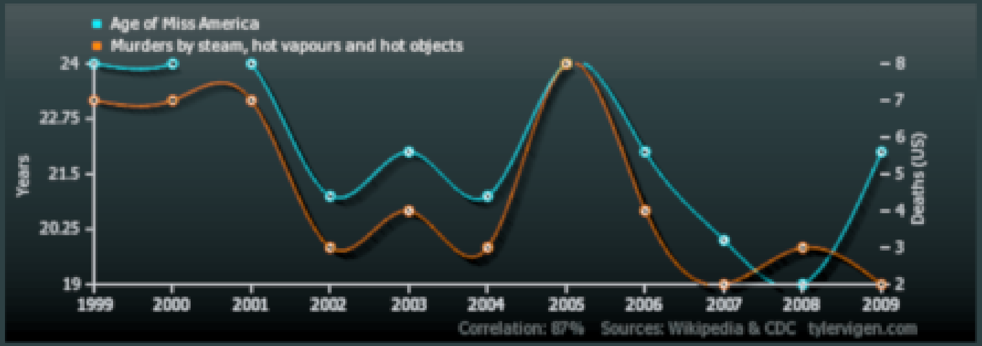
[2] x [3] = [2 3]

[3] x [2] = [3 2]

- Classical statistics often use a “joint-probability” mathematical space.
- This means that any 2 variables can be plotted against each other, irrespective of how silly the claim of a correlation might be.
- Quantum mechanics work in vector spaces.
- Instead of having precise points, there are vectors which give direction.
- These vectors are described mathematically by matrices.
- Using vector spaces would seem to be a very natural approach for Psychology.
- Psychologists often don’t want precise data about people, rather they are interested in how things like beliefs and attitudes might be changing.

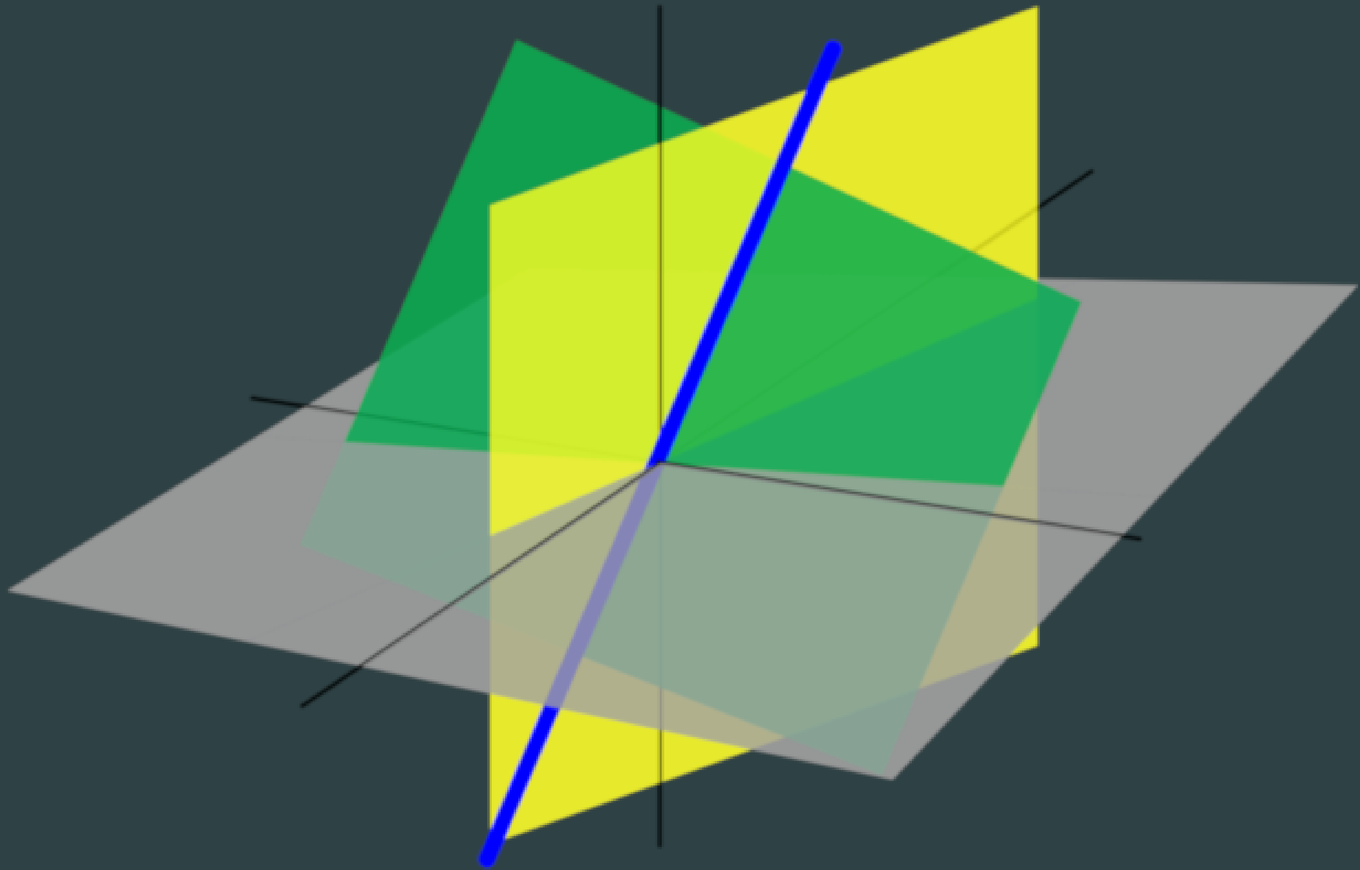
Correlation Plots





Source: www.tylervigen.com

Vector Space



Source: wikipedia.org

- Quantum mechanics relies upon a set of equivalences.
- The metric system is also based on a set of equivalences where 1 litre of water weighs exactly 1 kilogram and occupies exactly 1 cubic metre of space.
- These equivalences allow the way in which things are looked at to be changed very easily.
- For example, in Quantum mechanics it is possible to easily change from a wave function to probability by taking the square of a wave function’s amplitude. This is called “Born’s Rule” [doi:10.1126/science.122.3172.675].
- These equivalences might be just as useful in Psychology as they are in Physics.

Quantum Mechanical Equivalences

Matrix mechanics

is the same as

wave function mechanics

which transforms into both

probability spaces

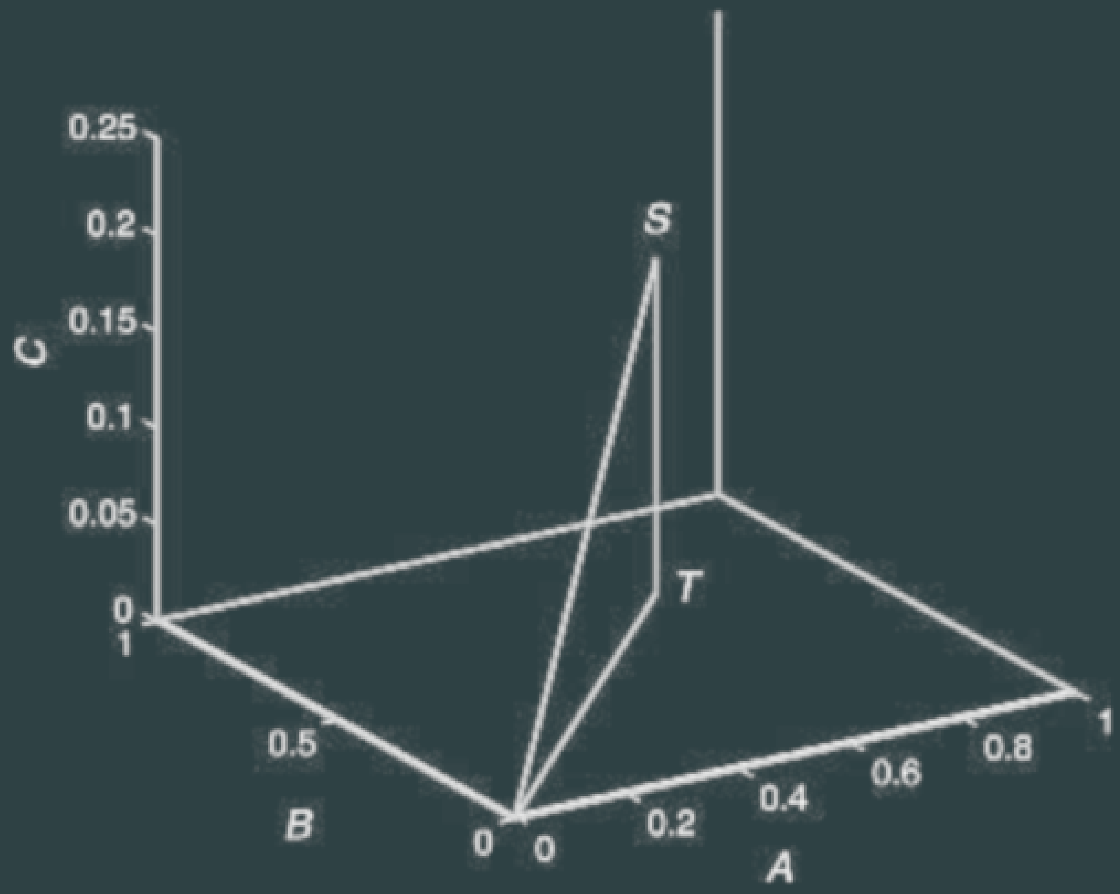
and

physical, sub-atomic spaces


Applying
Quantum
Cognition

- Busemeyer & Bruza [doi:10.1017/CBO9780511997716] suggest that ordering effects can be explained by quantum mechanics.
- They show that in a complex vector space, the position of the initial "state vector" is affected by the order in which items are considered.
- This change of position may affect the decision made.

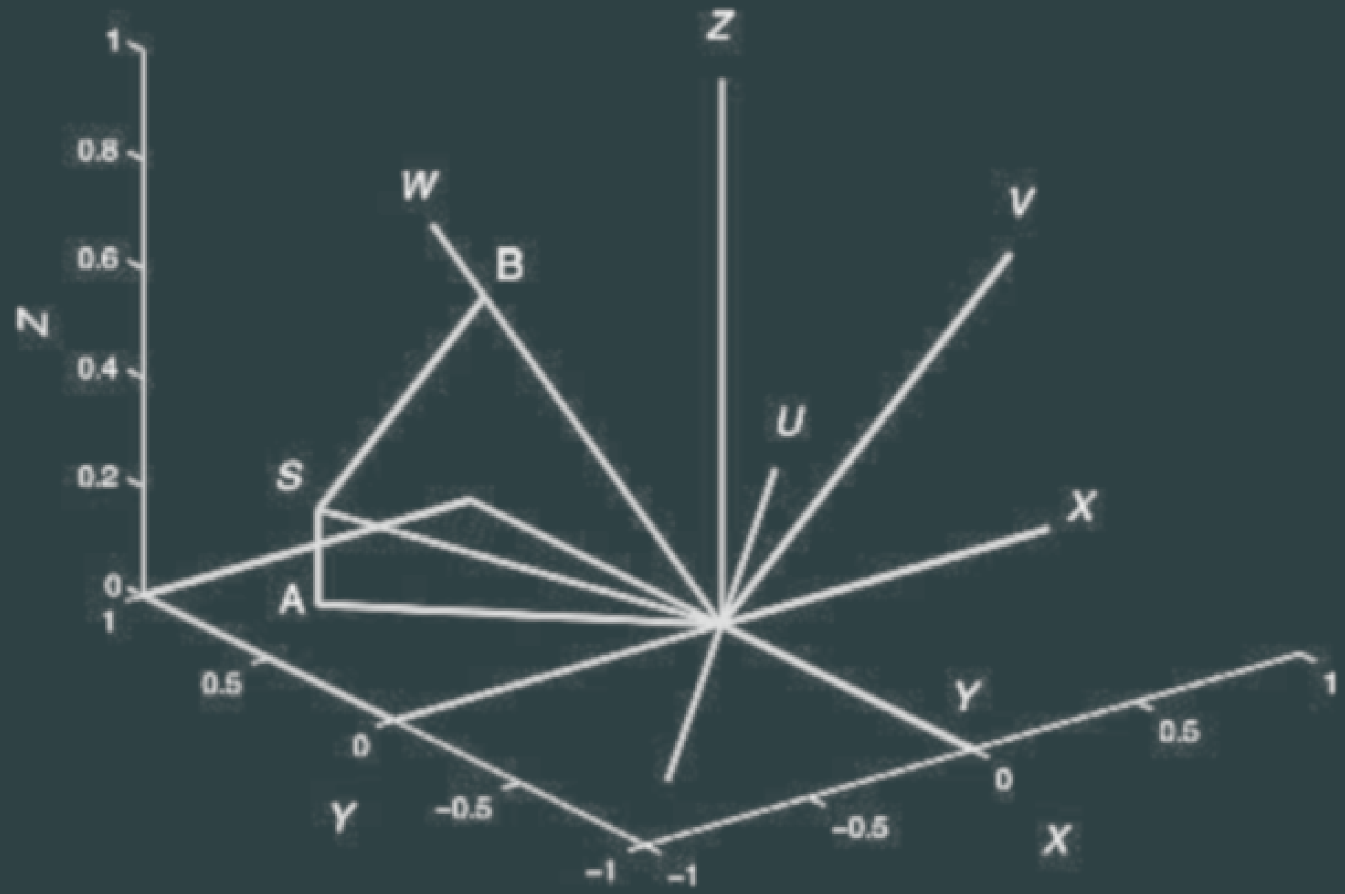
Quantum Ordering Effects



Three-dimensional real vector space: each orthogonal axis represents one car; *S* is the preference state, *T* is the projection on the *A*,*B* plane.

Source: Busemeyer & Bruza (2012)

Quantum Ordering Effects



Three-dimensional vector space with two different sets of base vectors: {*X*,*Y*,*Z*} and {*U*,*V*,*W*}. The initial state is *S*. *A* and *B* are projections.

Source: Busemeyer & Bruza (2012)

- The “Prisoner’s Dilemma” is an exercise in game-play theory.
- It is also a problem that occurs in the real world.
- The classic scenario is that the Police have arrested 2 people, but do not know which of them committed a crime.
- The prisoners are in separate rooms and unable to communicate with each other.
- If both prisoners stay silent they will both serve 1 year in prison.
- If both prisoners say that the other committed the crime, then they will both serve 2 years in prison.
- If one prisoner says that the other committed the crime, and the other stays silent, then the betrayer will walk free while the other will serve 3 years in prison.
- If the decision that the other prisoner is likely to make is ignored, it is always best for the prisoners to betray each other. This is the “dominant” outcome.
- However, if the other prisoner’s decision is taken into consideration, then it is better for both prisoners to stay silent since that leads to the “least worst” outcome.
- This dilemma results from the tension between the “Pareto Optimal” outcome and the “Nash Equilibrium”.
- Flitney & Abbott [doi:10.1142/S0219477502000981], amongst others, have proposed a quantum version of the Prisoner’s Dilemma.

- If the choices faced by each prisoner are entangled then there exists a form of communication between them.
- The ability to communicate changes the dominant outcome.
- Different degrees of entanglement will lead to different dominant outcomes.
- When 2 people are negotiating, it is hard to think of one person's strategy separately from the other's since both will respond to the behaviour of the other.
- This could be loosely thought of as a form of "entanglement", and might suggest that the quantum model of the Prisoner's Dilemma is more appropriate for real-world modelling than the classical version.

Quantum Prisoner's Dilemma

Prisoner A \ Prisoner B	Stays silent	Betrays other
Stays silent	Both get 1 year	Prisoner A: 3 years Prisoner B: Free
Betrays other	Prisoner A: Free Prisoner B: 3 years	Both get 2 years

- Bayes’ theorem is a way to calculate the likelihood of an event.
- The simplest version of Bayes’ theorem, the “Naïve Bayes’ Classifier”, calculates likelihood by multiplying the marginal probabilities of events — but this assumes that the data is “conditionally independent”. In other words, that the occurrence of one datum does not affect another.
- When the data is “conditionally dependent” other formulae may be used, such as the Expectation-Maximisation algorithm of Dempster, Laird, & Rubin [Jstor:2984875].
- What if whether the data is conditionally independent, or conditionally dependent, is unknown?
- Bond, He, & Ormerod [doi:10.1142/S0219749918500028] have addressed this problem using quantum mechanics.
- A quantum mechanical “isomorphic representation” of contingency data allows any question which might be asked to be answered through a simple rearrangement of terms.
- An “isomorphic representation” is a model which is mathematically identical to the original, and keeps all the ordinal data while preserving the internal relationships that exist between them.
- Experimental results demonstrate that the Quantum Bayes’ expression is a better model of human likelihood estimation than the Naïve Bayes’ Classifier.

Classical Bayes' Theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Manufacturer	Cadbury	Nestlé
Total number of sweets	10	10
% red sweets	0.8	0.7
% striped sweets	0.6	0.5

$$P(\text{Cadbury}|\text{Red, Striped}) = \frac{0.5 \cdot 0.8 \cdot 0.6}{(0.5 \cdot 0.8 \cdot 0.6) + (0.5 \cdot 0.7 \cdot 0.5)} = 0.578$$

Source: Bond, He, & Ormerod (2018)

Quantum Bayes' Theorem

$$P(H_\alpha | D_1 \cap D_2 \ldots \cap D_m) = \frac{\sum_{i,j} \sqrt{x_{i\alpha} x_{j\alpha}} c_{ij}^{\alpha}}{\sum_{i,j,\beta} \sqrt{x_{i\beta} x_{j\beta}} c_{ij}^{\beta}}$$

Manufacturer	Cadbury	Nestlé
Total number of sweets	10	10
% red sweets	0.8	0.7
% striped sweets	0.6	0.5

$$P(\text{Cadbury}|\text{Red}, \text{Striped}) = \frac{0.5 \cdot (0.8 + 0.6 + \frac{0.8 \cdot 0.6}{0.7 \cdot 0.5})}{(0.5 \cdot (0.8 + 0.6 + \frac{0.8 \cdot 0.6}{0.7 \cdot 0.5})) + (0.5 \cdot (0.7 + 0.5 + \frac{0.7 \cdot 0.5}{0.8 \cdot 0.6}))} = 0.5896$$

Source: Bond, He, & Ormerod (2018)

Quantum Bayes' Theorem

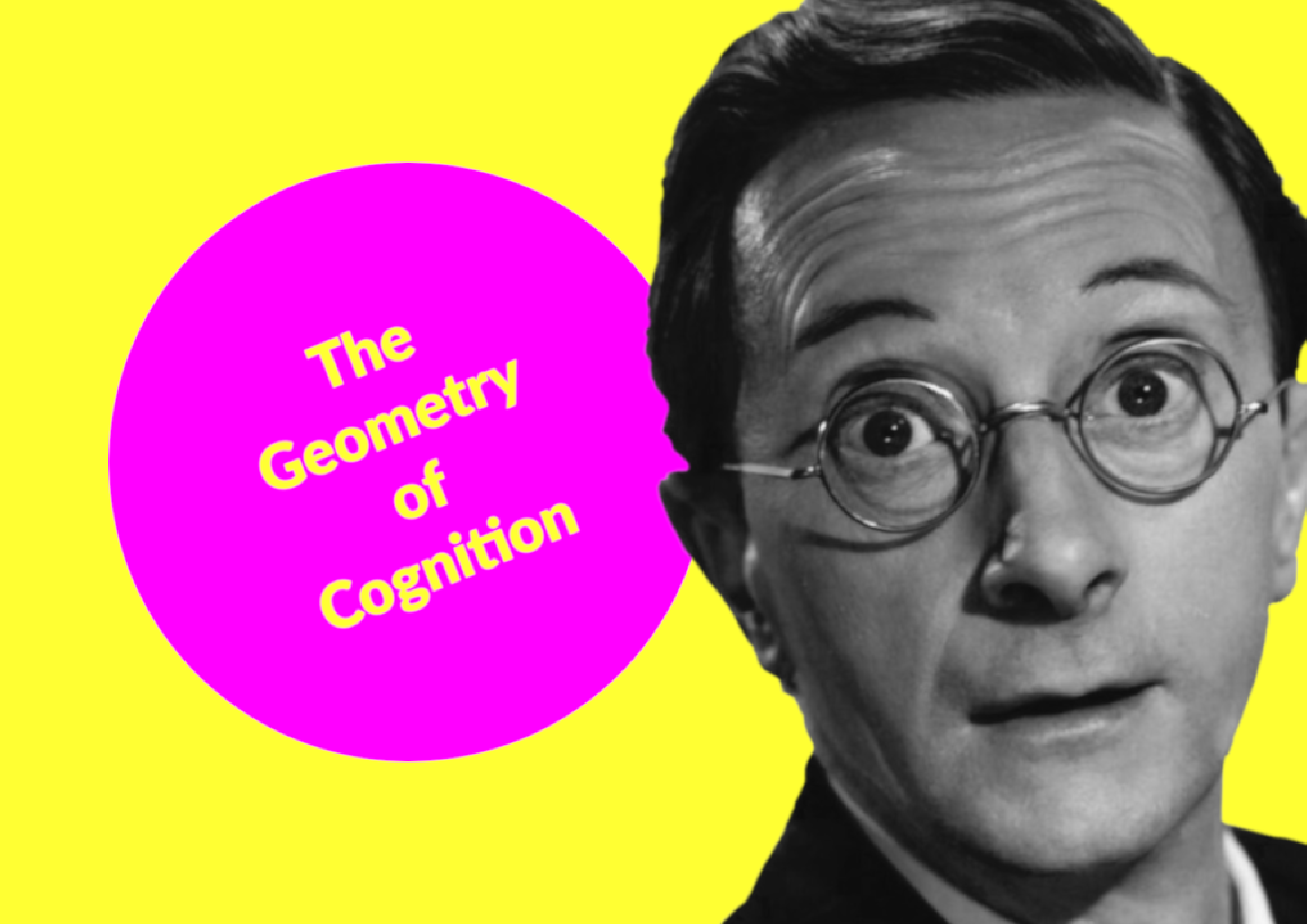
$$P(H_\alpha|D_1 \cap D_2 \ldots \cap D_m) = \frac{\sum\limits_{i,j} \sqrt{x_{i\alpha} x_{j\alpha}} c_{ij}^{\alpha}}{\sum\limits_{i,j,\beta} \sqrt{x_{i\beta} x_{j\beta}} c_{ij}^{\beta}}$$

is rewritten from

$$|\Psi\rangle = \frac{\sum_{\alpha,k} \sqrt{x_{k\alpha}} |H_\alpha \otimes D_k\rangle}{\sqrt{\sum_{i,j,\alpha} \sqrt{x_{i\alpha} x_{j\alpha}} c_{ij}^{\alpha}}}$$

which is a full, isomorphic mathematical representation of any sized contingency table, and includes all ordinal data and their relationships.

Source: Bond, He, & Ormerod (2018)


The
Geometry
of
Cognition

- Conceptualising cognition as something that acts upon information demands a level of algorithmic modelling which is almost certainly impossible to achieve.
- Flipping that proposition, however, gives a model in which information creates cognition, and that is both non-algorithmic and non-deterministic.
- Information has the necessary internal relationships to provide the mathematical structure for cognition through its geometry, but this can only be expressed by using quantum mechanical formalisms.
- Experimental data suggests than when people search for information it is not to identify a discrete datum, but to establish the relationships between data.
- Using quantum mechanics it is possible to quantify those relationships, even when values cannot be determined using classical mathematics.
- Such a model can, in part, explain dual-process theory, with apparent “fast” and “slow” processes merely being the difference in time taken to resolve relationships with high entropy, or degrees of freedom, as opposed to those with low entropy.
- Non-monotonicity assumes an algorithmic process. Without an algorithm, non-monotonicity may be seen to be the result of non-determinism.
- A geometric model allows the generation of an initial solution in NP (“non-deterministic, polynomial time”) problems, and for them to be solved.
- The quantum “equivalences” allow a geometric model of cognition to be evaluated mathematically.

The end